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Fast and Reliable Modeling of Piezoelectric Transducers for Energy Harvesting Applications

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Abstract

This paper presents a fast and reliable model identification technique for piezoelectric transducers based on an equivalent electromechanical circuit easily implementable on SPICE-like simulation tools. Model parameter extraction is simple and requires just standard and inexpensive laboratory equipment. The equivalent circuits of four cantilevered Q220-A4-303YB transducers from Piezo Systems with different tip masses were identified and several simulations were performed with different electric loading conditions; measurement results agree with model response predictions in all the experiments. Indeed, the equivalent circuit representation permits the evaluation of the response of a real harvester system, where the electronic load is a synchronized switching converter which usually causes a significant feedback on the mechanical part of the system during energy extraction. Both simulation and measurements show that the damping effect is particularly important near resonance, where the adopted model is able to fit the experimental data and provides a more realistic description of the behavior of the system.

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Keywords: piezoelectric devices; energy harvesting; equivalent electromechanical model; model identification;

1. Introduction

The present interest on pervasive sensor networks and the steady development of electronic devices with low power consumption motivates the research on electronic systems capable of harvesting energy from the surrounding environment. Among many viable methods, the use of piezoelectric devices for converting vibrations into electric energy seems particularly effective. However, a correct estimation of the harvester performance is important to reliably evaluate in an early phase of system design the joint

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performance of the piezoelectric transducer loaded by the devised power conversion circuit. This is particularly true when using synchronized switching converters [1-3]. Since such circuits are intrinsically non-linear and are triggered at specific time instants by smart control logic, the ability of simulating their performance with specific transducers is essential for system optimization. For this purpose, analytical or FEM models of piezoelectric devices produce quite accurate results but cannot be easily coupled to Spice simulations. On the other hand, many works on non-linear power conversion circuits are often based on excessively simplified assumptions, such as for example weak electromechanical coupling or out of resonance operation [1, 2]: these assumptions are particularly limiting in energy harvesting applications where mechanical structures are purposely excited near their resonant frequencies.

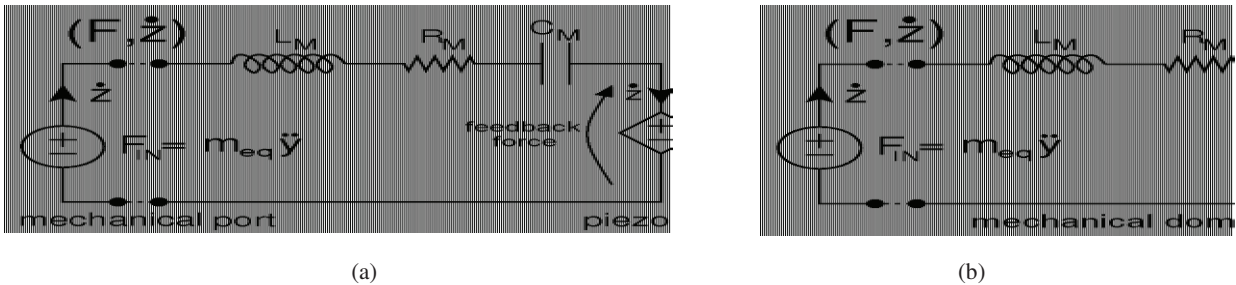


Fig. 1. (a) Equivalent electromechanical circuit of a piezoelectric transducer; (b) open circuit reduced equivalent network

2. Equivalent circuit of a piezoelectric transducer

Almost any piezoelectric transducer can be successfully modeled with a lumped elements circuit of the type illustrated in Fig. 1a and fully described in [4]. The circuitual approach is based on the electro-mechanic analogy: currents are associated to mass velocities \dot{z} and voltages are associated to forces; therefore, the model consists of two domains, the mechanical and the electrical one, interacting by means of a couple of controlled generators, as shown in Fig. 1a. In the mechanical part (left side) the inductor L_M accounts for the equivalent inertial mass, the capacitor C_M is related to the equivalent mechanical compliance and the resistor R_M models the structural losses associated to the specific mode of oscillation of the mechanical structure. This description works fairly well in the neighborhood of the resonant frequencies of the oscillating modes of interest. The mechanical part is fed by the force generator F opposed to a feedback force generator controlled by the voltage developed on the device output capacitor C_P (inverse piezoelectric effect). At the same time the mechanical velocity produces a forward current feeding both the output capacitor (direct piezoelectric effect) and any electric load connected to the transducer. Hence, model identification involves the following independent six parameters: L_M , C_M , R_M , C_P , α and β . In addition, it is sometimes handy to pull back the electrical impedance towards the mechanical domain, as illustrated in Fig. 1b.

2.1. Experimental setup and available measurements

The modeled transducers are a set of Q220-A4-303YB piezoelectric beams from Piezo Systems with additional tip masses respectively of 6 g, 10 g, 12 g and 18 g (Fig. 1a). The transducers are mounted on an inexpensive custom shaker system based on a Ciare CW200Z woofer, providing the impressed vibration force F_{IN} with arbitrary waveforms (Fig. 2b), while a Kionix KXP-84 accelerometer measures the actual input acceleration \ddot{y} applied to the base of the cantilever device. A further mechanical quantity, the peak to peak displacement Δz of the tip mass during an AC steady-state sinusoidal regime, is measured by taking a shot with a 1 s exposure time with a standard digital camera and comparing the pixels swept by

the tip elongation with those in a known length, as shown in Fig. 2c. Finally, an Agilent 34401A digital multimeter and an oscilloscope gauge the open circuit output voltage and the short circuit current. However, since common multimeters are unable to resolve very low AC current values (such those produced in a shorted device), a load resistor $R_L = 98.2 \, \Omega$ has been used to replace current acquisitions with voltage ones: the admittance of R_L is at least two order of magnitude higher than that of C_P at the frequencies of interest: R_L emulates the short circuit and almost cancels the mechanical feedback αV_P .

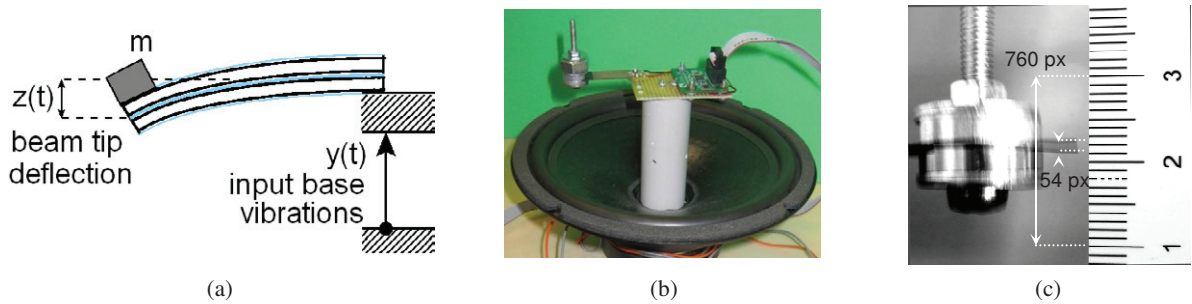


Fig. 2. (a) A cantilever piezoelectric transducer; (b) the experimental set-up; (c) a digital picture for evaluating tip displacement

Table 1. Summary of the measurements required for model extraction

Acquired parameter	In which experiment?	How is it determined?
τ damping time constant	Pulse mechanical excitation; open circuit load	The shaker applies a mechanical pulse; exponentially damped oscillations are analyzed with an oscilloscope
f_M mech. resonance frequency	Swept sinusoidal excitation; pseudo-short circuit configuration	$V_P(f)$ and $\ddot{y}(f)$ are measured. The shaker sweeps frequency until a maximum of $V_P(f)/\ddot{y}(f)$ is found
f_O open circuit peak frequency	Swept sinusoidal excitation; open circuit configuration	$V_P(f)$ and $\ddot{y}(f)$ are measured. The shaker sweeps frequency until a maximum of $V_P(f)/\ddot{y}(f)$ is found
$V_P(f_O)$ output voltage peak amplitude	Steady-state sinusoidal excitation at f_O ; open circuit configuration	The digital multimeter reads the AC voltage while the shaker applies the sinusoidal acceleration
$\Delta z(f_O)$ peak-to-peak tip displacement	Steady-state sinusoidal excitation at f_O ; open circuit configuration	A digital camera takes a 1s shot of the beam tip with a background ruler. Lengths are measured on pixel basis.
$\ddot{y}(f_O)$ impressed base acceleration	Steady-state sinusoidal excitation at f_O ; open circuit configuration	The accelerometer RMS is readout while the shaker applies the sinusoidal acceleration
C_P transducer electrical capacitance	Electrical excitation; clamped device condition	LCR-meter measurement at 10kHz: sinusoidal excitation well above cut-off of any important vibration mode

In this work, all the forces acting on the system are expressed in scaled units by assuming a unity equivalent mass m_{eq} . The measurements used for model identification can be classified according to the electrical loading condition (short or open) and to the type of mechanical excitation (pulse or sinusoidal). With sinusoidal excitations it is easy to sweep the frequency and detect the resonance peak. In a shorted configuration, the detected peak is at the true natural frequency f_M of the pure mechanical mode, when the piezoelectric feedback is totally cancelled. In an open circuit configuration, the resonance frequency f_O is shifted with respect to f_M because: (1) the effect of the feedback force generator αV_P and of the electrical capacitance C_P results in a capacitance C_{EQ} instead of C_M ; (2) the peak of a low-pass response is a function of the resonator damping time constant τ . Equations (1) clarify this aspect, whereas (2) expose the method:

$$f_M = \frac{1}{2\pi} \frac{1}{\sqrt{L_M C_M}}; \quad f_O = \frac{1}{2\pi} \sqrt{\frac{1}{L_M C_{EQ}} - \frac{2}{\tau^2}}; \quad \frac{1}{C_{EQ}} = \frac{1}{C_M} + \frac{\alpha\beta}{C_P}. \quad (1)$$

$$\beta = \frac{2C_P \cdot V_P(f_o)}{\Delta z(f_o)}; \quad R_M = \frac{\ddot{y}_{IN}(f_o)}{\Delta z(f_o) \cdot \pi f_o}; \quad L_M = \frac{\tau}{2} R_M; \quad C_M = \frac{1}{(2\pi f_M)^2 L_M}; \quad \alpha = \frac{C_P}{\beta} \left(\frac{1}{C_{EQ}} - \frac{1}{C_M} \right). \quad (2)$$

2.2. Experimental validation

In a first set of experiments (Fig. 3a) the identified models accurately predict the frequency response of the piezoelectric transducers with different masses and loading conditions. In a second set of experiments the synchronized charge extractor described in [3] was connected to the transducers. In case of sinusoidal input vibrations, during operation the circuit applies a periodic series of current pulses for tapping energy. At resonance this action results in a periodic force opposed to input vibrations damping the actual output voltage V_P^* down to half its nominal value (Fig. 3b) of twice the unloaded voltage V_P as explained in [3].

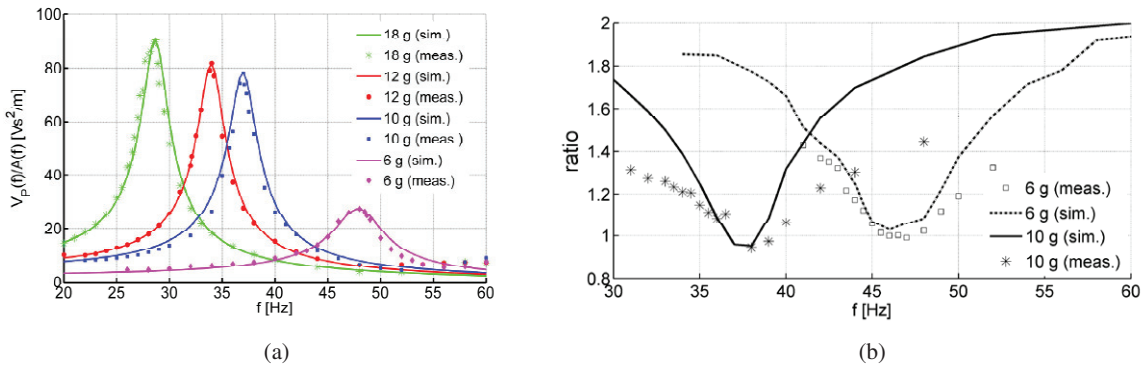


Fig. 3. (a) Comparison of measured and simulated frequency responses; (b) Voltage is measured in open circuit configuration (V_P) and when the power converter is activated (V_P^*). The plot shows the measured and simulated ratios V_P^*/V_P at different frequencies.

3. Conclusion

The presented modeling technique allows a circuit designer to quickly and reliably evaluate the loading effect of the non-linear power converter on the piezoelectric transducer and to readily estimate overall system performance. The loading effect cannot be neglected for small sized cantilever beams with tip inertial masses when forcing base vibrations have frequency components close to resonance.

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